第二章练习题

一. 填空

1. 523194687 的逆序数是11,它是奇排列.　　2. 若$126i48k97$ 为奇排列,则$i=5,k=3$.

3. $\tau(246\cdots(2n)(2n-1)(2n-3)\cdots 31)=n^2$, $\tau(135\cdots(2n-1)246\cdots(2n))=\dfrac{n(n-1)}{2}$.

4. 若排列 $j_1j_2\cdots j_{n-1}j_n$ 与排列 $j_nj_{n-1}\cdots j_2j_1$ 有相同的奇偶性,则$n=4k,4k+1$.

5. 设$4k+1$级排列 $j_1j_2\cdots j_{4k+1}$是奇排列,则 $j_{4k+1}\cdots j_2j_1$是奇排列,添上数码$4k+2$后构成的

$4k+2$级排列$(4k+2)j_{4k+1}\cdots j_2j_1$是偶排列.

6. 如果排列$x_1x_2\cdots x_{n-1}x_n$的逆序数是k,排列$x_nx_{n-1}\cdots x_2x_1$的逆序数是$\dfrac{n(n-1)}{2}-k$.

7. $|A|=|a_{ij}|$为4级行列式,项$a_{14}a_{22}a_{31}a_{43}$的符号是正,$a_{13}a_{41}a_{34}a_{22}$的符号是正.

8. 写出四级行列式中带负号且含有因子$a_{23}a_{31}$的项$-a_{14}a_{23}a_{31}a_{42}$.

9. n阶行列式D等于零的充要条件是D的某两行(或两列)的元素成比例或者D中一定有一行(或列)的元素全为零.此命题是否正确: 否.

10. 设n阶行列式D的值为c,若将D的所有元素都乘上-1,得到的行列式的值为$(-1)^n c$.

11. 设行列式$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=1$,$\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}=2$,则$\begin{vmatrix} a_1 & b_1+c_1 \\ a_2 & b_2+c_2 \end{vmatrix}=3$.

12. 已知$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=4$, 那么$\begin{vmatrix} a_{32} & a_{33} & a_{31} \\ a_{22} & a_{23} & a_{21} \\ a_{12} & a_{13} & a_{11} \end{vmatrix}=-4$.

13. 已知$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}=2$,则$\begin{vmatrix} z_1 & x_1 & y_1 \\ z_2 & x_2 & y_2 \\ z_3 & x_3 & y_3 \end{vmatrix}=2$,$\begin{vmatrix} x_1 & 2x_2 & x_3 \\ 3y_1 & 6y_2 & 3y_3 \\ -z_1 & -2z_2 & -z_3 \end{vmatrix}=-12$.

14. 设$D=\begin{vmatrix} 1 & -1 & 1 & 2 \\ 2 & 1 & 0 & -1 \\ -2 & 2 & 3 & 4 \\ -1 & 2 & -2 & -3 \end{vmatrix}$,$A_{ij}$为$(i,j)$元素的代数余子式,则$-2A_{11}+2A_{12}+3A_{13}+4A_{14}=0$.

15. 设$D=\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$,$A_{ij}$为$(i,j)$元素的代数余子式,求$\sum_{i=1}^{4}\sum_{j=1}^{4}A_{ij}=-4$

16. 在行列式$\begin{vmatrix} a & 1 & 2 \\ 0 & 4 & -1 \\ 3 & b & 1 \end{vmatrix}$中,$b$的代数余子式为$-24$,则$a=-24$.

17．已知$\begin{vmatrix}1&3&a\\1&2&0\\a&-1&4\end{vmatrix}$中代数余子式$A_{13}=7$,则代数余子式$A_{31}=8$.

18. 四阶行列式的第三行的元素为$-1,0,2,4$,第四行元素的代数余子式分别是$2,10,a,4$,则$a=-7$.

19. 四阶行列式的第三行的元素为$-1,2,-2,4$,其对应的余子式分别为$-5,3,-2,0$,则行列式等于3.

20. 设$f(x)=\begin{vmatrix}x&1&1&2\\1&x&1&-1\\3&2&x&1\\1&1&2x&1\end{vmatrix}$,则展开式中$x^3$的系数为$-1$.

21．齐次线性方程组$\begin{cases}x_1+x_2+x_3=0\\\lambda x_1+x_2=0\\x_1+\lambda x_2=0\end{cases}$有非零解的充要条件是$\lambda=1$或者$\lambda=-1$.

22. 齐次线性方程组$\begin{cases}x_1+x_2+x_3=0\\x_1+2x_2+ax_3=0\\x_1+4x_2+a^2x_3=0\end{cases}$有非零解,则$a=1$或$2$.

23. 若$\begin{vmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{vmatrix}=1$,则线性方程组$\begin{cases}a_{11}x_1-a_{12}x_2+b_1=0\\a_{21}x_1-a_{22}x_2+b_2=0\end{cases}$的解为$\begin{cases}x_1=-(b_1a_{22}-b_2a_{12})\\x_2=b_2a_{11}-b_1a_{21}\end{cases}$.

24. 多项式$\begin{vmatrix}1&2&3&4\\1&3-x^2&3&4\\3&4&1&12\\3&4&2&x^2+3\end{vmatrix}$的四个根为$\pm1,\pm3$, $\begin{vmatrix}1&1&1&1\\1&2&4&8\\1&-2&4&-8\\1&x&x^2&x^3\end{vmatrix}=0$的根为$1,2,-2$,

25. 多项式$\begin{vmatrix}1&1&1&\cdots&1\\1&1-x&1&\cdots&1\\1&1&2-x&\cdots&1\\\vdots&\vdots&\vdots&&\vdots\\1&1&1&\cdots&(n-1)-x\end{vmatrix}$的根为$0,1,2,\cdots,n-2$.

1) $$\begin{vmatrix}3&1&-1&2\\-5&1&3&-4\\2&0&1&-1\\1&-5&3&-3\end{vmatrix}=\begin{vmatrix}1&1&-2&3\\0&2&3&-3\\2&0&1&-1\\1&-5&3&-3\end{vmatrix}=\begin{vmatrix}1&1&-2&3\\0&2&3&-3\\0&-2&5&-7\\0&-6&5&-6\end{vmatrix}=\begin{vmatrix}1&1&-2&3\\0&2&3&-3\\0&0&8&-10\\0&0&14&-15\end{vmatrix}=40,$$

2) $$\begin{vmatrix}1&\cdots&1&1\\0&\cdots&2&1\\\vdots&&\vdots&\vdots\\n&\cdots&0&1\end{vmatrix}=\begin{vmatrix}1&\cdots&1&1-\sum_{k=2}^{n}\frac{1}{k}\\0&\cdots&2&0\\\vdots&&\vdots&\vdots\\n&\cdots&0&0\end{vmatrix}=(-1)^{\frac{n(n-1)}{2}}\left(1-\sum_{k=2}^{n}\frac{1}{k}\right)n!,$$

$$\begin{vmatrix} 1 & 1 & \cdots & 1 & 2-n \\ 1 & 1 & \cdots & 2-n & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ 2-n & 1 & \cdots & 1 & 1 \end{vmatrix}_n = \begin{vmatrix} 1 & 1 & \cdots & 1 & 2-n \\ 0 & 0 & \cdots & 1-n & n-1 \\ \vdots & \vdots & & \vdots & \vdots \\ 1-n & 0 & \cdots & 0 & n-1 \end{vmatrix}_n = \begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 1-n & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 1-n & 0 & \cdots & 0 & 0 \end{vmatrix}_n = (-1)^{\frac{n(n-1)}{2}}(1-n)^{n-1}$$

4) $$\begin{vmatrix} a+x_1 & a & a & \cdots & a \\ a & a+x_2 & a & \cdots & a \\ a & a & a+x_3 & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ a & a & a & \cdots & a+x_n \end{vmatrix} = \prod_{i=1}^{n} x_i \begin{vmatrix} 1+\frac{a}{x_1} & \frac{a}{x_2} & \frac{a}{x_3} & \cdots & \frac{a}{x_n} \\ -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{vmatrix} = \prod_{i=1}^{n} x_i \left(1+\sum_{i=1}^{n}\frac{a}{x_i}\right),$$

5) $D = \begin{vmatrix} 0 & b & b & \cdots & b \\ a & 0 & b & \cdots & b \\ a & a & 0 & \cdots & b \\ \vdots & \vdots & \vdots & & \vdots \\ a & a & a & \cdots & 0 \end{vmatrix}$,每个位置都加 x 得行列式为 D_1,则 $D_1(x) = D + xt$

$D_1(-a) = D - at = (-a)^n, D_1(-b) = D - bt = (-b)^n$,则 $(a-b)D = a(-b)^n - b(-a)^n$.

6) $$D = \begin{vmatrix} 1 & x_1 & x_1^{\ 2} & \cdots & x_1^{\ n} \\ 1 & x_2 & x_2^{\ 2} & \cdots & x_2^{\ n} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^{\ 2} & \cdots & x_n^{\ n} \\ 2-2 & -2 & -2 & \cdots & -2 \end{vmatrix} = \begin{vmatrix} 1 & x_1 & x_1^{\ 2} & \cdots & x_1^{\ n} \\ 1 & x_2 & x_2^{\ 2} & \cdots & x_2^{\ n} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^{\ 2} & \cdots & x_n^{\ n} \\ 2 & 0 & 0 & \cdots & 0 \end{vmatrix} + \begin{vmatrix} 1 & x_1 & x_1^{\ 2} & \cdots & x_1^{\ n} \\ 1 & x_2 & x_2^{\ 2} & \cdots & x_2^{\ n} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^{\ 2} & \cdots & x_n^{\ n} \\ -2 & -2 & -2 & \cdots & -2 \end{vmatrix}$$

$$= (-1)^n 2\left(\prod_{i=1}^{n} x_i\right)V + (-2)V\left(\prod_{i=1}^{n}(1-x_i)\right) = 2V\left[(-1)^n\left(\prod_{i=1}^{n} x_i\right) - \left(\prod_{i=1}^{n}(1-x_i)\right)\right]$$

7) $$\begin{vmatrix} a & -1 & 0 & \cdots & 0 & 0 \\ ax & a & -1 & \cdots & 0 & 0 \\ ax^2 & ax & a & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ ax^{n-2} & ax^{n-3} & ax^{n-4} & \cdots & a & -1 \\ ax^{n-1} & ax^{n-2} & ax^{n-3} & \cdots & ax & a \end{vmatrix} = \begin{vmatrix} a & -1 & 0 & \cdots & 0 & 0 \\ 0 & a+x & -1 & \cdots & 0 & 0 \\ 0 & 0 & a+x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a+x & -1 \\ 0 & 0 & 0 & \cdots & 0 & a+x \end{vmatrix} = a(a+x)^{n-1}$$

8) $$\begin{vmatrix} 1 & a_1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1-a_1 & a_2 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1-a_2 & a_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1-a_{n-1} & a_n \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1-a_n \end{vmatrix} = \begin{vmatrix} 1 & a_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & a_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & a_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & a_n \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = 1,$$

9) $\begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2 & x & -1 & \cdots & 0 & 0 \\ a_3 & 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x & -1 \\ a_n & 0 & 0 & \cdots & 0 & x \end{vmatrix} = \begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2+a_1x & 0 & -1 & \cdots & 0 & 0 \\ a_3+a_2x+a_1x^2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1}+a_{n-2}x+\cdots & 0 & 0 & \cdots & 0 & -1 \\ f(x) & 0 & 0 & \cdots & 0 & 0 \end{vmatrix} = f(x)$,

10) $\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & x+1 & 3 & \cdots & n \\ 1 & 2 & x+1 & \cdots & n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 2 & 3 & \cdots & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ 0 & x-1 & 0 & \cdots & 0 \\ 0 & 0 & x-2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x-n+1 \end{vmatrix} = \ldots$,

11)

$$\begin{vmatrix} a_{11} & 1 & a_{12} & 1 & \cdots & a_{1n} & 1 \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ a_{21} & x_1 & a_{22} & x_2 & \cdots & a_{2n} & x_n \\ x_1 & 0 & x_2 & 0 & \cdots & x_n & 0 \\ a_{31} & x_1^2 & a_{32} & x_2^2 & \cdots & a_{3n} & x_n^2 \\ x_1^2 & 0 & x_2^2 & 0 & \cdots & x_n^2 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & x_1^{n-1} & a_{n2} & x_2^{n-1} & \cdots & a_{nn} & x_n^{n-1} \\ x_1^{n-1} & 0 & x_2^{n-1} & 0 & \cdots & x_n^{n-1} & 0 \end{vmatrix} = \pm \begin{vmatrix} a_{11} & 1 & a_{12} & 1 & \cdots & a_{1n} & 1 \\ a_{21} & x_1 & a_{22} & x_2 & \cdots & a_{2n} & x_n \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & x_1^{n-1} & a_{n2} & x_2^{n-1} & \cdots & a_{nn} & x_n^{n-1} \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ x_1 & 0 & x_2 & 0 & \cdots & x_n & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-2} & 0 & x_2^{n-2} & 0 & \cdots & x_n^{n-2} & 0 \\ x_1^{n-1} & 0 & x_2^{n-1} & 0 & \cdots & x_n^{n-1} & 0 \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} & 1 & \cdots & 1 & 1 \\ a_{21} & \cdots & a_{2n} & x_1 & \cdots & x_{n-1} & x_n \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} & x_1^{n-1} & \cdots & x_{n-1}^{n-1} & x_n^{n-1} \\ 1 & \cdots & 1 & 0 & \cdots & 0 & 0 \\ x_1 & \cdots & x_n & 0 & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-2} & \cdots & x_n^{n-2} & 0 & \cdots & 0 & 0 \\ x_1^{n-1} & \cdots & x_n^{n-1} & 0 & \cdots & 0 & 0 \end{vmatrix}$$

12) $\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ n+1 & n+2 & n+3 & \cdots & 2n \\ 2n+1 & 2n+2 & 2n+3 & \cdots & 3n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (n-1)n+1 & (n-1)n+2 & (n-1)n+3 & \cdots & n^2 \end{vmatrix}$.

2. 求行列式$D=\left|a_{ij}\right|_n$,其中(1) $a_{ij}=i+j$. (2) $a_{ij}=ij$. (3) $a_{ij}=i+j-ij$. (4) $a_{ij}=\max\{i,j\}$.

3. 问λ,μ取何值时,齐次线性方程组$\begin{cases} \lambda x_1+x_2+x_3=0 \\ x_1+\mu x_2+x_3=0 \\ x_1+2\mu x_2+x_3=0 \end{cases}$有非零解.

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 2\mu & 1 \end{vmatrix} = \begin{vmatrix} \lambda-1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & \mu & 0 \end{vmatrix} = -(\lambda-1)\mu=0$$

4. 用克拉默法则解线性方程组$\begin{cases} x_1+x_2+x_3+x_4=5 \\ x_1+2x_2-x_3+4x_4=-2 \\ 2x_1-3x_2-x_3-5x_4=-2 \\ 3x_1+x_2+2x_3+11x_4=0 \end{cases}$.

5. 设 $a_1, a_2, \cdots, a_n$ 为数域 P 中两两不等的数,求线性方程组 $\begin{cases} x_1 + a_1x_2 + a_1^2x_3 + \cdots + a_1^{n-1}x_n = 1 \\ x_1 + a_2x_2 + a_2^2x_3 + \cdots + a_2^{n-1}x_n = 1 \\ \cdots \\ x_1 + a_nx_2 + a_n^2x_3 + \cdots + a_n^{n-1}x_n = 1 \end{cases}$ 的解.

三 证明题

1．证明 $D_n = \begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & \cdots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \cdots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha+\beta & \alpha\beta \\ 0 & 0 & 0 & \cdots & 1 & \alpha+\beta \end{vmatrix} = \begin{cases} (n+1)\alpha^n & \alpha=\beta \\ \dfrac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta} & \alpha\neq\beta \end{cases}$.

2．设 $n \geq 2$，证明 $\begin{vmatrix} 1 & 2 & 3 & 4 & \cdots & n \\ 1 & 1 & 2 & 3 & \cdots & n-1 \\ 1 & x & 1 & 2 & \cdots & n-2 \\ 1 & x & x & 1 & \cdots & n-3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & x & x & x & \cdots & 1 \end{vmatrix} = (-1)^{n+1}x^{n-2} \quad (x \neq 0)$.

3. 证明: $\begin{vmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{vmatrix} = (a+b+c+d)(a+b-c-d)(a-b+c-d)(a-b-c+d)$.

证明: $\begin{vmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{vmatrix} = (a+b+c+d)\begin{vmatrix} 1 & b & c & d \\ 1 & a & d & c \\ 1 & d & a & b \\ 1 & c & b & a \end{vmatrix} = (a+b+c+d)\begin{vmatrix} 1 & b & c & d \\ 0 & a-b & d-c & c-d \\ 0 & d-b & a-c & b-d \\ 0 & c-b & b-c & a-d \end{vmatrix}$

$= (a+b+c+d)\begin{vmatrix} a-b & d-c & c-d \\ d-b & a-c & b-d \\ c-b & b-c & a-d \end{vmatrix} = (a+b+c+d)(a+d-b-c)\begin{vmatrix} a-d & b-c \\ b-c & a-d \end{vmatrix}$

$= (a+b+c+d)(a+b-c-d)(a-b+c-d)(a-b-c+d)$.

4. 证明: $\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2$.

证明: $\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix}\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix} = \begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix}\begin{vmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{vmatrix} = \begin{vmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t & 0 \\ 0 & 0 & 0 & r \end{vmatrix}$

其中 $(t=a^2+b^2+c^2+d^2)$,则 $D^2=(a^2+b^2+c^2+d^2)^4$,故 $D=\pm(a^2+b^2+c^2+d^2)^2$.

行列式的展开式中 a^4 的次数为1,故 $D=(a^2+b^2+c^2+d^2)^2$.

5. 设 $\alpha\neq k\pi$,证明 n 级行列式 $\begin{vmatrix} 2\cos\alpha & 1 & 0 & \cdots & 0 & 0 \\ 1 & 2\cos\alpha & 1 & \cdots & 0 & 0 \\ 0 & 1 & 2\cos\alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2\cos\alpha & 1 \\ 0 & 0 & 0 & \cdots & 1 & 2\cos\alpha \end{vmatrix}=\dfrac{\sin(n+1)\alpha}{\sin\alpha}$.

利用第一题的结果. $D_n=2\cos\alpha D_{n-1}-D_{n-2}$,取 α,β 满足 $\alpha+\beta=2\cos\alpha,\alpha\beta=1$,即 $x^2-2\cos\alpha x+1=0$ 的两个根. $x=\dfrac{2\cos\alpha\pm\sqrt{4\cos^2\alpha-4}}{2}=\cos\alpha\pm i\sin\alpha$,代入第一题的结论中可得结果.

6. 证明 $D_{2n}=\begin{vmatrix} a_n & & & & & b_n \\ & \ddots & & & \iddots & \\ & & a_1 & b_1 & & \\ & & c_1 & d_1 & & \\ & \iddots & & & \ddots & \\ c_n & & & & & d_n \end{vmatrix}=\prod_{i=1}^{n}(a_id_i-b_ic_i)$.