2017-2018 学年第一学期月考 2 行列式

$4i21j \quad 43215 \quad 43, 42, 41, 32, 31, 21$

一、填空题

1. 当 $i=$ __5__ $, j=$ __3__ 时，5 阶行列式的项 $a_{14}a_{2i}a_{32}a_{41}a_{5j}$ 取负号.

2. 四阶行列式 $\left|a_{ij}\right|_4$ 的展开式中含有因子 a_{23} 的项的个数是 __6__.

3. $\begin{vmatrix} 2017 & 2015 \\ 2016 & 2014 \end{vmatrix} =$ __$\begin{vmatrix} 1 & 1 \\ 2016 & 2014 \end{vmatrix}$__. $= \begin{vmatrix} 0 & 1 \\ 2 & 2014 \end{vmatrix} = -2$

4. 9 阶反对称行列式的值为 __0__.

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & \lambda & 1 & 1 \\ 0 & 1 & \lambda & 1 \\ 0 & 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & \lambda-1 & 0 & 0 \\ -1 & 0 & \lambda-1 & 0 \\ -1 & 0 & 0 & \lambda-1 \end{vmatrix} = \left(1+\frac{3}{\lambda-1}\right)(\lambda-1)^3 = (\lambda+2)(\lambda-1)^2$$

5. 方程组 $\begin{cases} \lambda x_1 + x_2 + x_3 = 1, \\ x_1 + \lambda x_2 + x_3 = 1, \\ x_1 + x_2 + \lambda x_3 = 1 \end{cases}$ 有唯一解，则 λ 满足 __$\lambda \neq -2$ 且 $\lambda \neq 1$__.

6. $\begin{vmatrix} 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 2 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 2015 & \cdots & 0 & 0 & 0 \\ 2016 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 2017 \end{vmatrix} =$ __$(-1)^{\frac{2016\cdot 2015}{2}} 2017!$__. $= 2017!$

7. $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & xz & xy \end{vmatrix} =$ __$xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \end{vmatrix} = \frac{xyz}{xyz}\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$__. $= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (y-x)(z-x)(z-y)$

32
52, 54
72, 74, 76
$\cdots$
$(2n-1)2, \cdots, (2n-1)(2n-2)$
$1+2+3+\cdots+(n-1)$

8. 排列 $135\cdots(2n-1)246\cdots(2n)$ 的逆序数为 __$\frac{(n-1)n}{2}$__.

9. 排列 $x_1x_2\cdots x_9x_{10}$ 的逆序数是 k，则排列 $x_{10}x_9\cdots x_2x_1$ 的逆序数是 __$\binom{10}{2}-k$__. $= 45-k$

二、计算 n 阶行列式 $D_n = \begin{vmatrix} 2 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 0 & \cdots & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix}$.

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二、计算 n 阶行列式 $D_n = \begin{vmatrix} 2 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 0 & \cdots & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 2 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix}$.

解

加边法

法一

$$D_n = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 2 & 0 & \cdots & 1 & 1 \\ 0 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \cdots & 2 & 0 \\ 0 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 1 & -1 & \cdots & 0 & 0 \\ -1 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -1 & 0 & 0 & \cdots & 1 & -1 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1-n & 2 & 1 & \cdots & 1 & 1 \\ -n & 1 & 0 & \cdots & 0 & 0 \\ -(n-1) & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -2 & 0 & 0 & \cdots & 1 & 0 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = (1-n+2n+(n-1)+\cdots+2+1)$$

$$= 1+\frac{n(n+1)}{2}$$

法二 各行减最后一行

$$D_n = \begin{vmatrix} 1 & -1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & -1 & \cdots & 0 & -1 \\ 0 & 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & -1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \\ 1 & 2 & 3 & \cdots & n-1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 2 & 3 & \cdots & n-1 & \end{vmatrix} = \frac{n(n+1)}{2}+1$$

$2+1+2+\cdots+2(n-1)$

法三 拆项法

$$D_n = \begin{vmatrix} 2 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 0 & \cdots & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 0 & \cdots & 1 & 1 \\ 1 & 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 2 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix} = D_{n-1} + \begin{vmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$$

$$= D_{n-1} + \begin{vmatrix} 1 & 0 & & & & \\ & 1 & 0 & & & \\ & & 1 & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & 1 & 0 \\ 1 & 2 & 3 & \cdots & n-1 & n \end{vmatrix} = D_{n-1}+n$$

$$= n+n-1+\cdots+2+D_1 = 2$$

$$= \frac{n(n+1)}{2}+1$$

三、计算 n 阶行列式 $D_n=\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}$.

解

法一

$$D_n=\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 1 & \cdots & 1 & 1 \\ 0 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \cdots & 0 & 1 \\ 0 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 & & 1 & 1 \\ -1 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -1 & 0 & 0 & \cdots & -1 & 0 \\ -1 & 0 & 0 & \cdots & 0 & -1 \end{vmatrix}=(1-n)(-1)^n=(-1)^n(1-n)$$

法二

$$D_n=\begin{vmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}=\begin{vmatrix} -1 & 0 & & & & \\ & -1 & 0 & & & \\ & & -1 & \ddots & & \\ & & & \ddots & -1 & 0 \\ 1 & 2 & 3 & \cdots & n-1 & n-1 \end{vmatrix}=(-1)^{n-1}(n-1)$$

四、利用克拉默法则求解线性方程组 $\begin{cases} x_1+2x_2-x_3=2, \\ x_1-2x_2+2x_3=3, \\ 2x_1-x_2+x_3=3. \end{cases}$

解

$$D=\begin{vmatrix} 1 & 2 & -1 \\ 1 & -2 & 2 \\ 2 & -1 & 1 \end{vmatrix}=3$$

$$D_1=\begin{vmatrix} 2 & 2 & -1 \\ 3 & -2 & 2 \\ 3 & -1 & 1 \end{vmatrix}=3,\quad D_2=\begin{vmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{vmatrix}=6,\quad D_3=\begin{vmatrix} 1 & 2 & 2 \\ 1 & -2 & 3 \\ 2 & -1 & 3 \end{vmatrix}=9$$

$$x_1=\frac{D_1}{D}=1,\quad x_2=\frac{D_2}{D}=2,\quad x_3=\frac{D_3}{D}=3.$$

五、n 阶行列式 D 中每个数 a_{ij} 分别用 2^{i-j} 乘所得的行列式记为 D_1，求行列式 D_1 的值.

解

$$D_1=\begin{vmatrix} a_{11} & 2^{-1}a_{12} & 2^{-2}a_{13} & \cdots & 2^{1-n}a_{1n} \\ 2a_{21} & a_{22} & 2^{-1}a_{23} & \cdots & 2^{2-n}a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ 2^{n-1}a_{n1} & 2^{n-2}a_{n2} & 2^{n-3}a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

$$=2^{1+2+\cdots+n}\begin{vmatrix} a_{11} & 2^{-1}a_{12} & 2^{-2}a_{13} & \cdots & 2^{1-n}a_{1n} \\ a_{21} & 2^{-1}a_{22} & 2^{-2}a_{23} & \cdots & 2^{1-n}a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & 2^{-1}a_{n2} & 2^{-2}a_{n3} & \cdots & 2^{1-n}a_{nn} \end{vmatrix}$$

$$=2^{1+2+\cdots+n}\,2^{-1-2-\cdots-n}D$$

$$=D$$